



Annex to GEAR -1

Calculation of the resistance of the gear tooth

User manual

Symbology

B = a break

P = a surface pressure



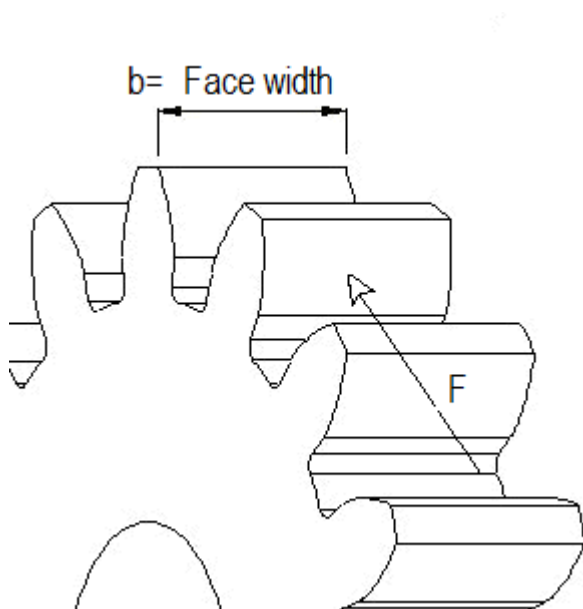
Symbol	Description	Unit	
b	Effective face width	mm	B-P
m_o	Normal module (tool)	mm	B
X_m	Profile shift (correction) on the beam	mm	-
x	Displacement coefficient		-
F	Tangential load	daN	B-P
Ω	Base tension correction coefficient	-	P
d1	Pitch diameter (Z1)	mm	P
Cr	Factor ratio	-	P
C_β	Factor helical teeth	-	P
K_v	Barth velociti Factor	-	B-P
K_{HL}	Factor pressure duration	-	P
K_M	Contact factor	-	B-P
K_A	Application factor	-	B-P
K_{BL}	Factor at break time	-	B
Y_ϵ	Factor of conduct (no contact ratio)	-	B
Y_F	Lewis form factor	-	B
Y_β	Factor helix angle	-	B
$\sigma_{b \text{ lim}}$	Stress tension limit at the base	daN/mm ²	B
σ_b	Stress tension at the base	daN/mm ²	B

Dimensioning of gear wheels with parallel axes.

Necessary to calculate the width of the toothing of the pinion and of the wheel gear at breaking and surface pressure.

The greater of these four values will be chosen as the face width of the two gears.

Sizing to surface stress. $\alpha_n = 20^\circ$



F

$$b = \frac{F}{\Omega_0 * d_1 * C_r * C_\beta * K_v * K_{HL} * K_M * K_A}$$

For $\alpha_n = 15^\circ$ multiply F for 0.92

For $\alpha_n = 17^\circ 30'$ multiply F for 0.96

For $\alpha_n = 25^\circ$ multiply F for 1.07

b= Face width (mm)

F = Tangential load (daN)

d_1 = Pitch diameter (mm)

The gear calculated with the above formula can bear an instantaneous overload up to $3 * F$ for a time of 15÷20 sec.

Given a power P to be transmitted (Kw) and a number of revolutions per minute (RPM)

It calculates the torque (torque) in daN * meter.

$$\text{Couple} = \frac{954.9 * P}{\text{RPM}} = (\text{daN} * \text{Mt}) \quad F = \frac{\text{Couple} * 2000}{d1} = \text{daN}$$

Q0 Material

0.1-0.2	Cast Iron
0.2-0.8	Steel untreated
0.8-1.3	Steel hardened surface induction
1-1.5	Steel hardened and tempered

C_r = Factor ratio (It 'the same for pinion and wheel)

$$i = \frac{Z2}{Z1}$$

$$\text{for external gear } C_R = \frac{i}{i + 1}$$

$$\text{for internal gear } C_R = \frac{i}{i - 1}$$

C_β = Factor helical teeth (It 'the same for pinion and wheel)

It depends on the helix angle β , $C_\beta = 1 + (0.0376 * \beta^{0.658})$

Trend to C_β

β°	C _β
0°	1
5°	1.10
10°	1.18
15°	1.24
20°	1.28
25°	1.32
30°	1.35
35°	1.38
40°	1.40

K_v = Barth velocity Factor (It 'the same for pinion and wheel)

V_p = Max : (20-25 m/1")

$$V_p = \frac{d_1 * \pi * \text{RPM}}{60000}$$

The gears are divided into 4 quality classes.

Class 1 - toothing of extreme precision for high speed gears (up and over 100 m / sec) finishing grinding.

Class 2 - Precision toothing (50 m / sec) finishing grinding.

Class 3 - Good quality (20 m / sec). without grinding

Class 4 - Mediocre quality (5 m / sec) without grinding

$$K_v = \begin{aligned} & \text{class 1} = \frac{30}{30 + \sqrt{V_p}} \\ & \text{class 2} = \frac{12}{12 + \sqrt{V_p}} \\ & \text{class 3} = \frac{6}{6 + \sqrt{V_p}} \\ & \text{class 4} = \frac{3}{3 + \sqrt{V_p}} \end{aligned}$$

K_{HL} = Factor pressure duration (Different from the pinion and wheel)

When the transmission has a variable duty cycle must take account of equivalent duration.

D_0 = equivalent duration

$C_1 - C_2 \text{-----} C_n$ = Different load

$D_1 - D_2 \text{-----} D_n$ = Durations related to the different loads $D_1 + D_2 \text{-----} D_n = D$ total duration.

The equivalent duration is expressed as a function of the maximum load that call C_1

$$D_0 = D_1 + D_2 * \left(\frac{C_2}{C_1} \right)^n + D_n * \left(\frac{C_n}{C_1} \right)^n$$

$n = 10$ for breaking calculation $= 6$ for surface pressure calculation

K_{HL} It is expressed in function of the number of cycles.

$$K_{HL} = 8.44 * n^{\circ} \text{ cycles}^{-0.13}$$

Trend to K_{HL}

K_{HL}	Numero cicli
2	10^5
1.45	10^6
1	10^7
0.7	10^8
0.5	10^9
0.5	10^{10}

N.B.

Nel caso di una corona dentata intermedia il numero di cicli deve essere moltiplicato per 2.

K_M = Contact factor (It 'the same for pinion and wheel)

It depends on the ratio $\frac{b}{d_1}$

b = Face width

d_1 = Pitch diameter of pinion (Z_1)

K_M	b/d_1
1	1
0.95	1.5
0.84	2
0.72	2.5

b being an unknown value can be set with the calculation $K_M = 1$ that is a valid value in almost all cases.

The b / d_1 ratio should not exceed the value of 2.

The absolute maximum is 2.5

K_A = Application Factor (It 'the same for pinion and wheel)

Operation without shock 1
 Operating moderate shock 2
 Operating with substantial shock 3

Power element	Operat. W. shock	12 hours / day	24 hours / day
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Electric motor	1	1	0.95
	2	0.8	0.7

Turbine	3	0.67	0.50
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Internal combustion engine single cylinder	1	0.8	0.7
	2	0.67	0.57
	3	0.57	0.45

Internal combustion engine multicylinder	1	0.67	0.57
	2	0.57	0.45
	3	0.45	0.35

For installations that require absolute security divide these values by $1.25 \div 1.4$

Sizing at Break (Bending stress)

$$b = \frac{F * Y_{\varepsilon} * Y_F * Y_{\beta}}{\sigma_{b \text{ lim}} * m_0 * K_V * K_{bL} * K_M * K_A}$$

b = Face width in mm

m_0 = normal module (tool)

F= Tangential load (daN.)

The gear calculated with the above formula can bear an instantaneous overload up to 2*F for a time of 15-20 sec.

Y_ϵ = Factor of conduct (no contact ratio) (It 'the same for pinion and wheel)

With class gears 1 and 2, the old standard included:

$$Y_\epsilon = \frac{1}{\epsilon_\alpha}$$

New formula I.S.O.: $Y_\epsilon = 0.25 + (0.75 / \epsilon_\alpha)$

With teeth of class 3, and with substantial shock , and teeth Class 4: **$Y_\epsilon = 1$**

ϵ_α = ratio of conduct (it is appropriate that it is above 1.30)

(relativ Z1) (relativ Z2)

$$\epsilon_\alpha = Y_1 * U_1 + Y_2 * U_2$$

$$Y = \frac{ha}{m} = \frac{\text{addendum}}{\text{normal module}} \quad \text{in normal teeth} = 1$$

U = coefficient dependent on Z1 and Z2 and the angle of the **operating** pressure **α'** .

α'

Z	12°30'	15°	17°30'	20°	22°30'	25°	27°30'	30°
10	0.8	0.75	0.7	0.68	0.64	0.62	0.6	0.6
15	0.9	0.8	0.75	0.72	0.69	0.67	0.64	0.62
20	0.95	0.85	0.80	0.77	0.73	0.69	0.66	0.65
25	1	0.9	0.83	0.8	0.75	0.71	0.68	0.67
30	1.05	0.95	0.87	0.83	0.78	0.73	0.69	0.68
40	1.1	1	0.9	0.87	0.8	0.75	0.7	0.69
50	1.15	1.05	0.95	0.89	0.82	0.77	0.71	0.7
70	1.2	1.1	1	0.9	0.84	0.79	0.72	0.72
100	1.3	1.15	1.03	0.92	0.85	0.8	0.73	0.72
150	1.35	1.18	1.05	0.95	0.88	0.81	0.75	0.73
250	1.4	1.2	1.1	0.97	0.9	0.81	0.75	0.73

Y_F = Lewis form factor (Different from the pinion and wheel)

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It is a function of the number of the teeth, possible correction, the angle of pressure α_0 , the radius of curvature at the bottom of the tooth, etc.

$Z_i = N \cdot \cos \beta$ imaginary teeth = $Z / (\cos \beta)^3$ for helical teeth

$Z_i = Z$ for straight toothings

X_m = profile on the beam displacement (correction in mm)

x = displacement coefficient = X_m / m_n

For $\alpha_0 = 20^\circ$, Y_F this is indicated in the table below.

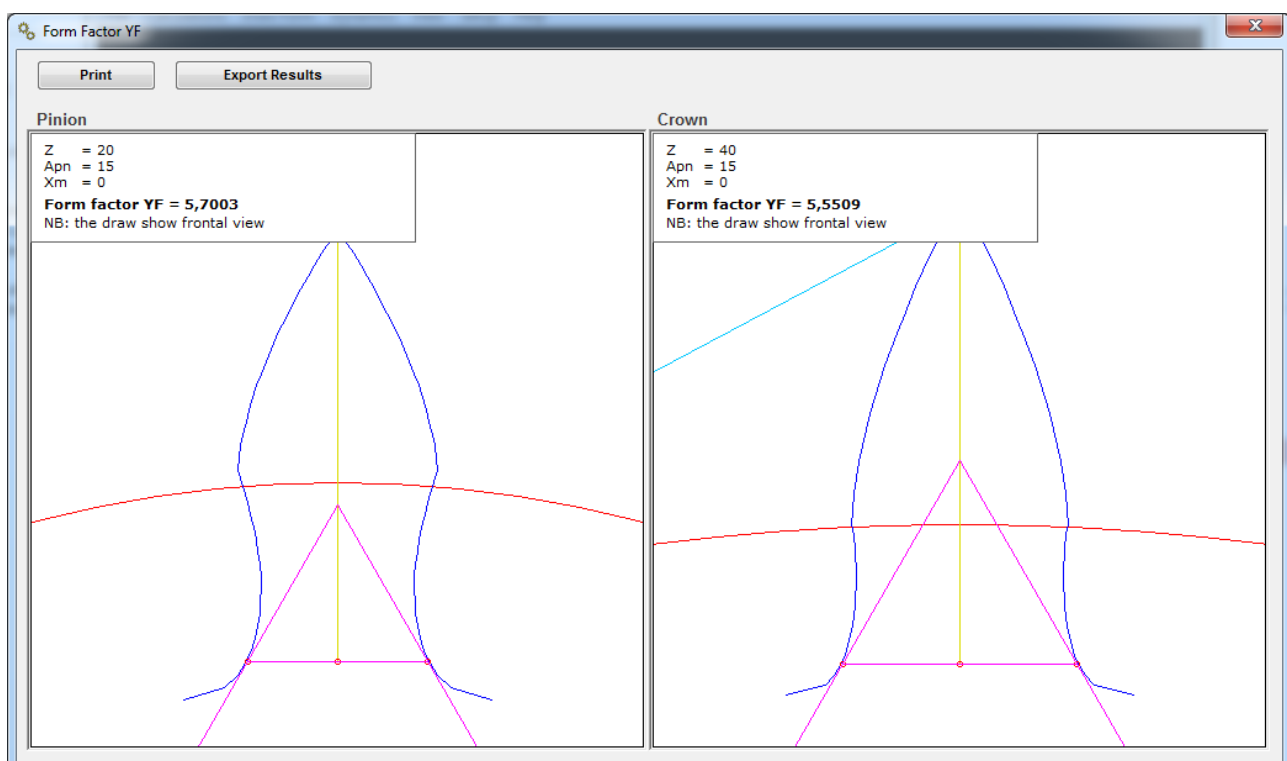
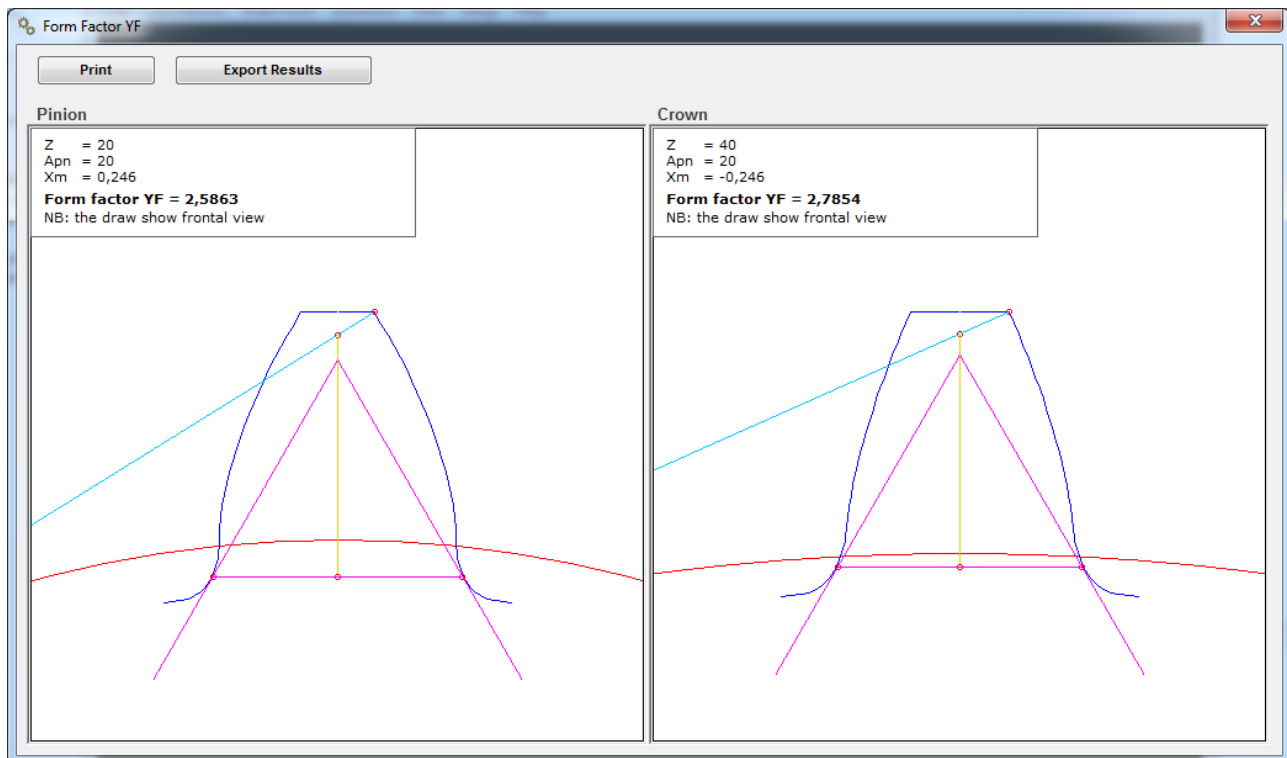
X											
Z_i	-0.5	-0.4	-0.3	-0.2	-0.1	0	+0.1	+0.2	+0.3	+0.4	+0.5
15	-	-	-	-	-	-	2.85	2.66	2.51	2.36	2.24
20	-	-	-	-	2.97	2.78	2.60	2.48	2.38	2.28	2.17
25	3.55	3.35	3.11	2.93	2.77	2.6	2.48	2.38	2.30	2.22	2.14
30	3.25	3.08	2.91	2.74	2.62	2.5	2.40	2.32	2.25	2.18	2.12
40	2.90	2.78	2.68	2.58	2.47	2.38	2.32	2.27	2.21	2.16	2.10
50	2.70	2.62	2.53	2.47	2.38	2.32	2.28	2.22	2.18	2.14	2.08
70	2.52	2.47	2.39	2.35	2.30	2.27	2.22	2.18	2.15	2.11	2.07
100	2.38	2.34	2.30	2.27	2.23	2.20	2.18	2.14	2.12	2.09	2.06
150	2.28	2.26	2.24	2.20	2.18	2.16	2.14	2.12	2.10	2.08	2.06
200	2.23	2.21	2.18	2.17	2.16	2.14	2.13	2.11	2.09	2.07	2.06
300	2.18	2.17	2.16	2.15	2.14	2.11	2.10	2.09	2.08	2.06	2.06
500	2.14	2.13	2.12	2.11	2.10	2.09	2.08	2.07	2.06	2.05	2.06

For $\alpha_0 = 15^\circ$ multiply for 1.22

For $\alpha_0 = 25^\circ$ multiply for per 0.848

“GEAR 2 CALCULATION”, calculates the Lewis form factor, whatever the profile of the teeth, even the most absurd that are not listed in the table.

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Y_β = Factor helix angle β (It 'the same for pinion and wheel)

Y_β	β
1	0°
0.93	5°
0.87	10°
0.82	15°
0.78	20°
0.76	25°
0.75	30°
0.75	35°
0.74	40°

$\sigma_{b \text{ lim}}$ = Stress tension at the base. (Bending stress)

It depends on the material of construction.

(in the case of an intermediate wheel multiply $\sigma_{b \text{ lim}}$ for $\frac{3}{4}$)

Material	σ_b
Gray irons	5-8
Special cast irons	8-12
Bronz	6-8
Carbon steels	12-18
Alloy steels	20-25
Alloy steels in total hardening	25-35
Alloy steel hardening	30-42

K_{bL} = Factor at break time (Different from the pinion and wheel)

It is expressed as a function of the number of cycles

K_{bL}	Number of cycles
1.6	10^5
1.25	10^6
1	10^7
0.8	10^8
0.65	10^9
0.65	10^{10}

N.B.

In the case of an intermediate wheel, the number of cycles must be multiplied by 2

After choosing the width of the toothed, It calculates the actual value stress.
Both for the pinion for the wheel.

$$\sigma_b = \frac{F_t}{b * m_0} * Y_\epsilon * Y_F * Y_\beta$$

Sample calculation

Number of pinion teeth $Z_1 = 30$

Wheel teeth number $Z_2 = 90$

pressure angle $\alpha = 20^\circ$

Module $m_0 = 2.5$

$d_1 = 75$

$d_2 = 225$

Number revolutions of the pinion = 2100

Drive with electric motor - Shocks moderate

Material of both the gears = 18 Ni Cr Mo 5 hardened and tempered

$\sigma_{blim} = 42$

$\Omega = 1.3$

Precision toothing finishing grinding. Class 2 ($V_p < 50$ m/sec.)

Duration = 20000 hours

Moderate shock =< 12 12 hours / day (Application Factor = 0.8)

Coppia sul pignone =

max 17 daN per il 30% della durata

media 12 daN per il 50% della durata

minima 5 daN per il 20% della durata

Sizing at Break (Bending stress)

$$b = \frac{F * Y_\epsilon * Y_F * Y_\beta}{\sigma_{b \text{ lim}} * m_0 * K_V * K_{bL} * K_M * K_A}$$

$$F = \frac{17 * 2000}{75} = 453 \text{ daN}$$

$$Y_1 = Y_2 = 1$$

$$\epsilon_\alpha = 1 * 0.83 + 1 * 0.917 = 1.747$$

New formula I.S.O. :

$$Y_\epsilon = 0.25 + (0.75 / \epsilon_\alpha) = 0.25 + (0.75 / 1.747) = 0.679$$

$$Y_F = 2.5 \text{ for pinion, } 2.2 \text{ for wheel}$$

$$Y_\beta = 1$$

$$v_p = \frac{\pi * 75 * 2100}{60000} = 8.24 \text{ m/sec}$$

$$K_v (\text{class 2}) = \frac{12}{12 + \sqrt{8.24}} = 0.8$$

When the transmission has a variable duty cycle must take account of equivalent duration.
 D_0 = equivalent duration

$$\text{Equivalent duration } D_0 \text{ at surface pressure} \\ 6000 + 10000(12/17)^6 + 4000(5/17)^6 = 7239 \text{ ore}$$

$$\text{Equivalent duration } D_0 \text{ at break} \\ 6000 + 10000(12/17)^{10} + 4000(5/17)^{10} = 6307 \text{ ore}$$

$$N^\circ \text{ cycles pinion} = \text{RPM} * D_0 * 60 = 2100 * 7239 * 60 = 912.144.000 \\ < 109, > 108 \quad K_{bL} \cong 0.65$$

$$N^\circ \text{ cycles wheel} = 912.144.000 * Z_1/Z_2 = 304.038.000 \\ < 109, > 108 \quad K_{bL} = 0.8$$

$$K_{bL} = 0.65 \text{ for pinion, } 0.8 \text{ for wheel}$$

$$K_M = 1$$

$$K_A = 0.8$$

$$b \text{ pinion} = \frac{453 * 0.679 * 2.5 * 1}{42 * 2.5 * 0.8 * 0.65 * 1 * 0.8} = 17.6$$

$$b \text{ wheel} = \frac{453 * 0.679 * 2.2}{42 * 2.5 * 0.8 * 0.8 * 1 * 0.8} = 12.6$$

Sizing to surface stress

$$b = \frac{F}{\Omega_0 * d_1 * C_r * C_\beta * K_v * K_{HL} * K_M * K_A}$$

$$C_r = \frac{3}{3 + 1} = 0.75$$

$$C_\beta = 1$$

$$K_{HL} = 8.44 * 912.144.000^{-0.13} = 0.577 \quad \text{for pinion}$$

$$K_{HL} = 8.44 * 304.038.000^{-0.13} = 0.666 \quad \text{for wheel}$$

$$b_{\text{pinion}} = \frac{453}{1.3 * 75 * 0.75 * 1 * 0.8 * 0.577 * 1 * 0.8} = 16.77$$

$$b_{\text{wheel}} = \frac{453}{1.3 * 75 * 0.75 * 1 * 0.8 * 0.666 * 1 * 0.8} = 14.53$$

So we have 4 values, in which we must choose the largest.

	At break	At surface pressure
b pinion	17.6	12.5
b wheel	12.6	14.5

It is necessary to have a same face width or exceeding 17.6 mm

Band recommended = 20

Which is a higher value of the four measurements.

Stress calculation to actual breaking

(Note: that is influenced by the Lewis factor)

$$453$$

$$\text{Pinion } \sigma_b = \frac{453}{20 * 2.5} * 0.679 * 2.5 * 1 = 15.38 \text{ daN/mm}^2$$

$$453$$

$$\text{Wheel } \sigma_b = \frac{453}{20 * 2.5} * 0.679 * 2.2 * 1 = 13.53 \text{ daN/mm}^2$$